Constructing Infinite Families of Solutions to Diophantine Equation $x^4 + y^4 + z^4 = 2n^2w^4$

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Abstract

This paper presents a systematic method to construct infinite families of primitive integer solutions to the quartic Diophantine equation $x^4 + y^4 + z^4 = 2n^2w^4$ for each square-free positive intger n with $\gcd(n, 290) = 1$. We identify this surface as a variant of the type **A217** in the classification by Bright [3], characterized by a high Néron-Severi rank of 4. Our method reduces the problem to system of two parameter-dependent quadratic equations, and subsequently to finding rational points of a quartic elliptic curve. By employing MAGMA's 4-descent to compute the Mordell-Weil group, we effectively realize the theoretical richness of rational points guaranteed by the surface's geometric structure. We demonstrate the effectiveness of this approach by providing non-trivial solutions for $n = 41, 101, 1007$. Furthermore, we propose a conjecture that non-trivial rational points exist for any square-free positive integer n with $\gcd(n, 290) = 1$, supported by extensive numerical evidence.

1 Introduction

The study of quartic Diophantine equations of the form $x^4 + y^4 + z^4 = kw^4$ has been central in number theory since Noam Elkies' landmark discovery of counterexamples to Euler's conjecture [1]. While Elkies found solutions to $x^4 + y^4 + z^4 = w^4$, the case with coefficient $2n^2$ on the right remains largely unexplored except for computational searches [9, 10, 11].

As pointed out by Elkies (private communication), the surface defined by is related to the case **A217** in the classification by Bright [3], which possesses a Néron-Severi rank of 4. This suggests a higher density of rational points compared to the original case **A197** studied by Elkies.

This paper provides the first parametrization of primitive solutions ($\gcd(x, y, z, w) = 1$ and without assuming any relation such as $z = x + y$ a priori) for several square-free n satisfying specific local conditions, via a novel reduction to elliptic curve rational points.

Using the above parameterization method, we have found infinite families of integer solutions to the Diophantine equation $x^4 + y^4 + z^4 = 2n^2w^4$ with $z \neq x + y$ for each $n = 33$, 41, 47, 51, 59, 69, 77, 83, 89, 101, 119, 123, 137, 141, 149, 159, 161, 173, 179, 187, 197, 209, 213, 227, 233, 249, 251, 253, 263, 267, 281, 287, 293, 299, 303, 311, 321, 323, 329, 339, 341, 353, 357, 383, 389, 393, 401, 407, 971, 977, 979, 983, 989, 1001, 1003, 1007 and 1013, for which no previously known solutions exist, in the range $n \leq 407$.

In this study, we verify the validity of the Hasse's principle for specific n and diagonal quartic surfaces $x^4 + y^4 + z^4 = 2n^2w^4$, and by using the 4-descent of MAGMA, we prove that the elements of the Tate-Shafarevich group actually correspond to rational points in concrete numbers.

2 Local Conditions

Consider primitive integer solutions (x, y, z, w) of the equation

$$x^4 + y^4 + z^4 = 2n^2w^4 \tag{1}$$

where n is a square-free positive integer and satisfies:

$$\begin{aligned} 2 &\nmid n, \\ 5 &\nmid n, \\ 29 &\nmid n. \end{aligned} \tag{2}$$

We can prove theorem of the local condition:

Theorem 1. If if n is square-free and $\gcd(n, 290) > 1$, then the equation (1) have no solutions with $\gcd(x, y, z, w) = 1$.

Proof: Trivial by checking $\pmod{16}$, $\pmod{5}$, and $\pmod{29}$.

3 Quadratic Curves and Simultaneous Equations

Assuming $z \neq 0$, normalize by dividing by z^4, and replace $\frac{x}{z}, \frac{y}{z}, \frac{w}{z}$ to x, y, t:

$$x^4 + y^4 + 1 = 2n^2t^4. \tag{3}$$

Introduce rational parameter u and two quadratic curves:

$$(u^2 - 2)y^2 = (-u^2 + 4u - 2)x^2 - 2(u^2 - 2u + 2)x + (-u^2 + 4u - 2) \tag{4}$$

$$\pm n(u^2 - 2)t^2 = (u^2 - 2u + 2)x^2 + (-u^2 + 4u - 2)x + (u^2 - 2u + 2). \tag{5}$$

The right-hand quadratic in (5) has no real roots, so sign is unique: $+n(u^2 - 2)t^2$ if $u^2 > 2$, $-n(u^2 - 2)t^2$ if $u^2 < 2$.

Theorem 2. Rational solutions (x, y, t) to (4),(5) yield solutions to (3).[5]

Proof: Trivial by simple calculation of rational expressions.

4 The Involution Map

For rational parameter u and variables x, y, t, we deine the following involution map τ:

$$\tau(u, x, y, t) = \begin{cases} (\frac{u-2}{u-1}, -x, y, t) & \text{if } u \neq 1, \\ (1, x, y, t) & \text{if } u = 1 \ . \end{cases} \tag{6}$$

The involution τ preserves the quadratic curves (4) and (5), and therefore all their common rational points (x, y, t) with except for the sign of x.

Therefore, the integer solutions of (1) are identical for parameter u and parameter $\tau(u) = \frac{u-2}{u-1}$. In other words, it is sufficient to find the common rational point (x, y, t) for either parameter u or parameter $\tau(u)$.

5 Reduction to Quartic Elliptic Curve

Given a constant H (for example, 200), using Helmut Hasse's local-global principle, we can select a rational number u such that the height $h(u) \leq H$, so that the two quadratic curves (4) and (5) each have a rational point. Then, we can filter the rational parameter u effectively. This contributes to the efficiency of rational point search.

For each rational parameter u, fix initial rational point (x_0, y_0) on (4); intersect with line

$$y = k(x - x_0) + y_0$$

to get second point $(x(k), y(k))$ as rational functions of k.

Substitute $x(k)$ into (5) to obtain the assoicated elliptic curve:

$$E_{n,u} : \pm Y^2 = aX^4 + bX^3 + cX^2 + dX + e, \tag{7}$$

where $Y = t(pk^2 + qk + r)^2$, $X = k$, and $p, q, r, a, b, c, d, e \in \mathbb{Z}$.

The right-hand quadratic in (5) has no real roots, so sign is unique: $+Y^2$ if $a > 0$, $-Y^2$ if $a < 0$.[6]

Because $a, b, c, d, e \in \mathbb{Z}$, a rational point (X, Y) on (7) with $X \notin \mathbb{Z}$ should be a non-torsion point .

6 Singular Cases

The first quadratic curve (4) is singular iff $u = 0, 1, 2$; then reduces to

$$x^2 + x + 1 = nt^2,$$

solvable only for $n = 1$. The second quadratic curve (5) is always nonsingular.[5]

Seiji Tomita and Oliver Couto [2] (Th.4.2) give a parameter solution to (1) using polynomials in p and q if there exist integers p and q such that $n = 3p^2 + q^2$. However, our parameter solution via the elliptic curves is different from this polynomial solution and we could apply even when $n \neq 3p^2 + q^2$.

7 Computing Rational Points with MAGMA

Transform (7) to Weierstrass form; apply MAGMA's `FourDescent` to compute rank, generators, and torsion points.

We choose a rational parameter u and the associated elliptic curve (7) with positive rank. For non-torsion point (X, Y) on the curve (7) , generate m-multiples; set $k = X$ and back-substitute to (x, y, t), then clear denominators for primitive (x, y, z, w).[6]

8 Necessity and Sufficiency of Local Conditions

We prove the main theorem:

Theorem 3. For arbitary square-free integer n, infinite primitive solutions of (1) exist iff $\gcd(n, 290) = 1$ and there exist a rational number u and the associated elliptic curve $E_{n,u}$ with positive rank .

Proof: Sufficiency via $rank E_{n,u} \geq 1$; necessity via p-adic obstructions for p=2,5,29.[4, 5]

Conjecture 1. (Weak conjecture)

If n is a square-free integer and $\gcd(n, 290) = 1$, then primitive solutions of (1) exist.

Remark 1. Conjecture 1 is valid for $n \leq 401$.

Conjecture 2. (Strong conjecture)

If n is a square-free integer and $\gcd(n, 290) = 1$, then $rank E_{n,u} > 0$ for some rational number u.

Remark 2. Conjecture 2 implies Conjecture 1, and Conjecture 2 is valid for $n \leq 393$.

9 Numerical Examples

We explain the method of finding integer solution of (1).

For example $n = 41$, we can filter rational number u with $height(u) \leq 200$ so that both quadratic curve (4) and (5) has a rational point. Then we can examine the following 144 rational numbers u:

$$u = -1, \frac{1}{5}, \frac{1}{17}, \frac{-1}{49}, \frac{1}{53}, \frac{3}{2}, \frac{4}{17}, \frac{-4}{45}, \frac{-4}{49}, \frac{4}{137}, \frac{7}{9}, \frac{8}{9}, \frac{8}{25}, \frac{-8}{45}, \frac{8}{81},$$
$$\frac{-8}{101}, \frac{9}{4}, 10, \frac{11}{2}, \frac{-11}{149}, \frac{12}{109}, \frac{-12}{185}, \frac{13}{53}, \frac{15}{17}, \frac{15}{113}, \frac{-16}{97}, \frac{17}{81}, \frac{19}{2}, \frac{19}{149}, \frac{-19}{193},$$
$$\frac{-20}{81}, \frac{20}{109}, \frac{21}{37}, \frac{21}{61}, \frac{-28}{153}, \frac{29}{49}, \frac{30}{13}, \frac{-32}{53}, \frac{-32}{117}, \frac{3}{16}, \frac{-37}{181}, \frac{40}{113}, \frac{42}{17}, \frac{-43}{153}, \frac{44}{169},$$
$$\frac{48}{193}, \frac{-51}{157}, \frac{-52}{73}, \frac{52}{137}, \frac{3}{16}, \frac{-55}{49}, \frac{-56}{89}, \frac{56}{145}, \frac{57}{109}, \frac{61}{113}, \frac{-61}{117}, \frac{-64}{49}, \frac{64}{153}, \frac{-64}{157}, \frac{-64}{169},$$
$$\frac{-67}{61}, \frac{-67}{81}, \frac{67}{101}, \frac{69}{20}, \frac{76}{113}, \frac{-76}{153}, \frac{-76}{193}, \frac{76}{197}, \frac{77}{109}, \frac{-79}{153}, \frac{-80}{149}, \frac{-84}{65}, \frac{-84}{101}, \frac{88}{117}, \frac{-88}{145},$$
$$\frac{89}{121}, \frac{91}{149}, \frac{-91}{181}, \frac{92}{157}, \frac{93}{40}, \frac{94}{49}, \frac{-95}{149}, \frac{98}{53}, \frac{99}{50}, \frac{100}{153}, \frac{101}{40}, \frac{-101}{117}, \frac{101}{121}, \frac{102}{53}, \frac{-104}{125},$$
$$\frac{105}{52}, \frac{-112}{117}, \frac{116}{121}, \frac{-116}{149}, \frac{-116}{173}, \frac{-119}{89}, \frac{119}{145}, \frac{-120}{101}, \frac{120}{169}, \frac{126}{5}, \frac{127}{153}, \frac{128}{145}, \frac{-128}{153}, \frac{-132}{97}, \frac{-132}{181},$$
$$\frac{135}{34}, \frac{138}{85}, \frac{141}{20}, \frac{141}{32}, \frac{145}{64}, \frac{146}{29}, \frac{147}{157}, \frac{150}{37}, \frac{-152}{197}, \frac{153}{32}, \frac{153}{104}, \frac{154}{73}, \frac{161}{52}, \frac{161}{169}, \frac{162}{17},$$
$$\frac{162}{113}. \frac{165}{52}, \frac{167}{10}, \frac{168}{173}, \frac{171}{26}, \frac{173}{181}, \frac{-176}{197}, \frac{177}{8}, \frac{-177}{193}, \frac{177}{197}, \frac{178}{5}, \frac{179}{26}, \frac{182}{101}, \frac{186}{73}, \frac{187}{197},$$
$$\frac{-188}{137}, \frac{-188}{185}, \frac{189}{8}, \frac{189}{128}, \frac{-196}{153}, \frac{-196}{181}, \frac{198}{89}, \frac{198}{125}, \frac{-199}{193}.$$

For $u = \frac{198}{125}$, we easily get a inital rational point $R(\frac{-15559}{420776}, \frac{775755}{420776})$ of the quadratic curve (4).

A line with a slope of k that passes through R intersects the quadratic curve (4) with another point $(x(k).y(k))$.

Then, we have following:

$$y = k\left(x + \frac{15559}{420776}\right) + \frac{775755}{420776}, \tag{8}$$
$$x(k) = \frac{61878143k^2 + 6170355270k - 8594866697}{-1673426152k^2 + 6005735848}, \tag{9}$$
$$y(k) = \frac{3085177635k^2 - 8372793090k + 11072351115}{-1673426152k^2 + 6005735848}. \tag{10}$$

From (9) and (5), we have followings:

$$\left\{17251816(3977k^2 - 14273)^2 t\right\}^2 = 2876442050708682153 49k^4 - 1436879617899897971340k^3 + 3997420003135110578878k^2 - 6003499258764516668940k + 4279348392455021754229. \tag{11}$$

We set

$$Y = 17251816(3977k^2 - 14273)^2 t, \tag{12}$$
$$X = k, \tag{13}$$

then we have following the associated elliptic curve

$$E_{41,u} : Y^2 = 287644205070868215349X^4 - 1436879617899897971340X^3 + 3997420003135110578878X^2 - 6003499258764516668940X + 4279348392455021754229. \tag{14}$$

We confirm the syzygy transforms an elliptic curve

$$E^0_{41,u} : y^2 = x^3 - 131533381367244164215248081536630341171740672x - 496946952622325121564854492450656958760656582555086171408919691264 \tag{15}$$

to the elliptic curve $E_{41,u}$.
The minimal standard model of $E^0_{41,u}$ is

$$E^2_{41,u} : y^2 = x^3 + x^2 - 3237629437155959990x - 1919096286007227296033058600. \tag{16}$$

Considering $E_{41,u}$ to 2-descent of $E^2_{41,u}$, we execute MAGMA's 4-descent, and get two independent rational points of $E^2_{41,u}$:

$$P_1\left(\frac{-174942651055478264675987}{244780606357156}, \frac{20891124644686233492499046337781371}{3829706654220268054696}\right),$$
$$P_2\left(\frac{-14294068250419325295136330646500671045398507}{10710488752832501604605089844433796}, \frac{5513451734757779144704564173941245101175055697275239743153358521}{1108444452616653816973134639524599540568006219348344}\right).$$

By rational transformation $[2524656, 2124629306112, 0, 0]^{-1}$. we get two independent rational points of $E^0_{41,u}$

$$
\begin{aligned}
&Q_1\Big(\frac{-278766212361028884962560363157615040}{61195151589289},\\
&\quad \frac{420221691703817813182687945550095986873781957856977 92}{47871333177753350683 7}\Big),\\
&Q_2\Big(\frac{-227771972256155311443678093027490811581048511781975 40800}{26776221882081254011512724611 08449},\\
&\quad \frac{110902215869769914514115310780866829416727353104149941718960019719938514270779 34592}{13855555657708172712164182994057494257100077741 8543}\Big).
\end{aligned}
\tag{17}
$$

By syzygy, we have some rational points (X, Y) of $E_{41,u}$ from rational points of $E^0_{41,u}$, and some $k = X$:

$$
\begin{aligned}
k =& \frac{30407075}{26279287}, \frac{945967865}{1248057599}, \frac{1588437129}{13721456335}, \frac{-1219384020107}{2053661177915},\\
& \frac{7238621986098507}{15401438000287805}, \frac{7238621986098507}{15401438000287805}, \frac{2523484525026218281}{616919842550050055}, \frac{2523484525026218281}{616919842550050055},\\
& \frac{7918680427205562475}{1555531906528806611}, \frac{7918680427205562475}{1555531906528806611}, \frac{20673647009724378359 5}{45530104805281077279 7}, \frac{206736470097243783595}{455301048052810772797},\\
& \cdots .
\end{aligned}
$$

From (9),(10),(14),(12).(13), we have a rational point (x, y, t) which satifies (3). We have an integer solution (x, y, z, w) with $\gcd(x, y, z, w) = 1$ which satisfies (1). Changing sign and replacing x, y, z so that $0 < x \le y \le z$, we have following equations:

$$
\begin{aligned}
&35401855^4 + 40865628^4 + 53562031^4 = 3362 \cdot 7822733^4,\\
&762983397233069 40^4 + 1443760980248375 17^4 + 3920970542732226 11^4 = 3362 \cdot 5174574560491060 7^4\\
&2263093427856490844456 5^4 + 218426009168410193424516^4 + 383430470008039234123883^4,\\
&= 3362 \cdot 516308699310629389191 59^4\\
&66362246478987836342552992091201664685210465639474020^4\\
&+ 96903091845971137820040019904874448516482466179139001^4\\
&+ 124994312898188111523490811243725150204657332635449303^4\\
&= 3362 \cdot 179838831232746850498471369045331511826771885565115 41^4.
\end{aligned}
$$

For $u = \frac{-52}{137}$, similarly, we have the associated elliptic curve

$$
\begin{aligned}
E_{41,u} : Y^2 =& 146130108644622127X^4 - 345859546045705 60X^3 + 75769621840114278X^2\\
& - 370651588684471816X + 306037888200539443
\end{aligned}
\tag{18}
$$

and rational numbers $k = X$:

$$\begin{aligned}k =&\frac{16427}{15371}, \frac{-74303}{29149}, \frac{1042559}{1271441}, \frac{-2229773}{1793951},\\ &\frac{9347149504299326044313779 77}{6515723679924634626694399 19}, \frac{-61892621095341520962149406917}{55717219579384353477735995 59},\\ &\frac{106933810848166875740529987 0337}{185501518562540879013154670 2081}, \frac{127733868542333204049618241 0519}{22341553755145147533400487211 89},\\ &\cdots,\end{aligned}$$

and integer solutions of (1)

$$\begin{aligned}&92988^4 + 185585^4 + 200711^4 = 3362 \cdot 30433^4\\ &314623956181553354273249972099755645960593522094013^4\\ &+ 338302329138750105233094179362897255579886004117266 21^4\\ &+ 589583937458783115292809314626850620888785885036010 20^4\\ &= 3362 \cdot 794457269293760593121754169671213590452097730843372 7^4\\ &\cdots.\end{aligned}$$

For $u = \frac{44}{169}$, similarly, we have the associated elliptic curve

$$\begin{aligned}E_{41,u} : Y^2 =&417877855555951071X^4 + 1299785550450804690X^3 + 1519574898215554612X^2\\ &+ 791044232470008790X + 154651753418222341\end{aligned} \tag{19}$$

and rational numbers $k = X$:

$$\begin{aligned}k =&\frac{-36225655093}{42531504399}, \frac{-737917016899}{985152753743}, \frac{-3284931787843}{315094317701}, \frac{-14199170625974}{19434669271167},\\ &\cdots.\end{aligned}$$

and integer solutions of (1)

$$\begin{aligned}&5441389686377966197^4 + 5952881649156946338 1^4 + 7370607112236248198 0^4\\ &= 3362 \cdot 1057663780912317296 7^4\\ &\cdots.\end{aligned}$$

Similarly, for several n we can find a rational number u, a initial rational point (x_0, y_0) of (4), a rational number k , and an integer solution (x, y, z, w) with $0 < x \leq y \leq z$, the result is **Appendix 2,3**.

10 Conclusion

This yields rational points (x, y, t) of (3) via Mordell-Weil groups of (7). If we could find a rational number u and the associated elliptic curve $E_{n,u}$ with $rankE_{n,u} > 0$, then the method systematically generates infinte families of primitive solutions of (1) .

While a universal parametric solution for all n remains an open question, the successful derivation of a 46-digit solution for $n = 101$ through our elliptic curve reduction framework marks a significant departure from previous constraints. Future work will focus on analyzing the height of these points to better understand the arithmetic complexity of solutions for $n \equiv 2 \pmod 3$.

The proposed algorithm is not limited to A_{217} surfaces but is also readily applicable to those of type A_{155} (Néron-Severi rank 3).

We can easily modify Elkies(Demjanenko) [1]'s second quadratic equation as follows.

$$2(x^4 + 6x^2y^2 + y4) + t^4 = 1 \tag{20}$$

$$(u^2 + 2)y^2 = -(3u^2 - 8u + 6)x^2 - 2(u^2 - 2)x - 2u \tag{21}$$

$$\pm n(u^2 + 2)t^2 = 4(u^2 - 2)x^2 + 8ux + (2 - u^2). \tag{22}$$

In fact, the effectiveness of this approach has been numerically verified for $n = 2, 5, 6, 10,$ $11, 13, 19, 23, 29, 30, 31, 51, 59,$ and 237, where we successfully generated explicit rational points in each case (see **Appendix 4**). For example, we find solutions for $n = 10, 11, 19, 51, 59$ which are non-congruent numbers, as follows:

$$261246816677719984125376675431014979^4 + 995355569908015340271785613176155930^4$$
$$+ 100 \cdot 421728649650332897209549903592601186^4 = 1427242172593177154199688515965361079^4,$$
$$\cdots .$$

$$80202538547017255548535317556474987176034903646049911^4$$
$$+ 357875720105384695686304974638016716956142041933914210^4$$
$$+ 100 \cdot 118825016210387182261744237492875690521504689721131194^4$$
$$= 436733538504200184127285271853259310131966303073062811^4,$$
$$\cdots .$$

$$6317639^4 + 41597960^4 + 121 \cdot 72383300^4 = 240122361^4,$$
$$\cdots .$$
$$22454117760^4 + 281831029555^4 + 361 \cdot 145773881796^4 = 641474481677^4,$$
$$\cdots .$$
$$230^4 + 735^4 + 2601 \cdot 152^4 = 1139^4,$$
$$\cdots .$$
$$6087^4 + 30320^4 + 3481 \cdot 3420^4 = 33913^4,$$
$$\cdots .$$

Acknowledgments

Thanks to Noam Elkies for his email response and the following suggestions:

"Martin Bright's doctoral thesis (2002, with the late Swinnerton-Dyer) <https://boojum.org.uk/maths/quartic-surfaces/thesis.pdf> *concludes an extensive appendix (App. A, pages* $128-153$*) that tabulates information including the rational Neron-Severi rank of every possible such surface* $aA^4+bB^4+cC^4+dD^4=0$ *over a number field* F*; for* $F=\mathbb{Q}$ *there are hundreds of cases. I see that your surfaces appear as* **A217** *(page* 138*, with the coefficients* $(1,-8c_1^2,1,1)$ *– I don't know why he didn't write it more simply as* $(1,1,1,-2c_1^2)$*); the rank is* 4*, where generically such surfaces have rank* 1*, even with three equal coefficients (* **A252** *on page* 141*); so in some sense it was known that this family could admit more elliptic fibrations than is typical, but again I don't know if anybody ever found your family of curves* E_u*, let alone found rational points on them, before you. (The Euler quartic* $(1,1,1,-1)$ *appears as* **A197** *on p.*137*, and also has rational Neron-Severi rank* 4*.)*

There are also Appendix B and C (pages $154-160$*) listing* (a,b,c,d) *for which the surface has no "small" solutions even though there is no local obstruction. But those all have two positive and two negative coefficients, where your coefficients have* 3 *of one sign and* 1 *on the other, so the lists in those Appendices do not bear on your work."*

The author is grateful to Noam Elkies for pointing out the connection to Martin Bright's classification $A217$ of diagonal quartic surfaces.

Appendix 1: MAGMA Implementation

```
// search rational points for 4-descent fd and bound M

SetClassGroupBounds("GRH");

function RP4(fd,M)
 T0:=Realtime();
 for J:=1 to #fd do
  printf "J="; J;
    FD:=fd[J];
    pts:=PointsQI(FD,M);
    F,m:=AssociatedEllipticCurve(FD); F;
    printf "rootno="; RootNumber(F);
    for K:=1 to #pts do
      P:=m(pts[K]); P; printf "height "; Height(P);
      IsPoint(F,P[1]);
    end for; //K
 end for; //J
 T1:=Realtime(T0);
 printf "realtime="; T1;
 return #fd;
```

```
end function;

// calculate 4-descent for considering y^2=f(x) as 2-descent of the syzygy
// elliptic curve of 4th degree polynomial f(x)

P<x> := PolynomialRing(Rationals());

function C0(f)
SetClassGroupBounds("GRH");
C := HyperellipticCurve(f);
time fd := FourDescent(C : RemoveTorsion);
#fd;
return fd;
end function;

//
// Example for elliptic curve :
//   E_{41,u}: Y^2 = 2876442050708682153 49*X^4 - 14368796178998979713 40*X^3  \
//                 + 3997420003135110578878*X^2 - 6003499258764516668940*X \
//                 + 4279348392455021754229
// Execute:
//
//   fd:=C0(2876442050708682153 49*x^4 - 14368796178998979713 40*x^3  \
//          + 3997420003135110578878*x^2 - 6003499258764516668940*x \
//          + 4279348392455021754229);
//
//   RP4(fd,10^12);
//
```

Appendix 2: Integer Solutions for $x^4 + y^4 + z^4 = 2n^2w^4$) ($1 \leq n \leq 407$, $971 \leq n \leq 1013$)

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \leq y \leq z)$	k	height(k)
1	$\frac{938}{241}$	$\left(\frac{1799}{1172}, \frac{1565}{1172}\right)$	(11270111669357, 22338600682595, 80267274165144, 67603989724187)	$\frac{30407075}{26279287}$	67.012
3	$\frac{-504}{4817}$	$\left(\frac{-2566}{1609}, \frac{4519}{1609}\right)$	(1609, 2566, 4519, 2257)	$\frac{-4519}{957}$	21.068
7	$\frac{3}{5}$	$\left(\frac{-1}{3}, \frac{2}{3}\right)$	(1, 2, 3, 1)	1	1.776
11	-1	$(1, -2)$	(19, 27, 50, 13)	$\frac{-1}{2}$	6.708
13	$\frac{-3}{5}$	$\left(\frac{-75}{103}, \frac{-284}{103}\right)$	(1, 4, 4, 1)	$\frac{-3}{5}$	1.450
17	$\frac{-7}{13}$	$\left(\frac{5}{3}, \frac{92}{51}\right)$	(2959, 7223, 15572, 3213)	$\frac{616}{323}$	16.659
19	$\frac{3}{5}$	$\left(\frac{-1}{3}, \frac{2}{3}\right)$	(25, 46, 63, 13), (40509, 120902, 2057435, 396911)	$\frac{3}{5}$, $\frac{-1223}{631}$	6.460, 26.871
21	$\frac{5}{13}$	$(-5, 4)$	(1, 4, 5, 1), (119023630860967209199, 119907594290190424115, 175046822661146704676, 35149498053729412309)	-1, $\frac{-5634785759}{8889499526}$	2.146, 89.896
23	-1	$(1, -2)$	(9, 29, 62, 11), (893, 1017, 1934, 349)	4, 30	7.124, 14.022
31	$\frac{-5}{13}$	$\left(\frac{1}{5}, \frac{-6}{5}\right)$	(1, 5, 6, 1)	-1	3.148
33	$\frac{24}{53}$	$\left(\frac{-309}{763}, \frac{100}{109}\right)$	(528988010581, 673826751736, 822834434251, 135897934731)	$\frac{14467817}{5083051}$	56.375
37	$\frac{-3}{101}$	$\left(\frac{20951}{17421}, \frac{5908}{17421}\right)$	(1223, 2612, 3709, 543)	$\frac{2491}{2084}$	15.517
39	-1	$(1, -2)$	(449, 653, 1210, 167)	$\frac{-8}{17}$	12.805
41	$\frac{198}{125}$	$\left(\frac{-15559}{420776}, \frac{775755}{420776}\right)$	(35401855, 40865628, 53562031, 7822733)	$\frac{2558149}{3980783}$	36.996
43	$\frac{7}{25}$	$\left(\frac{-841}{2179}, \frac{-2370}{2179}\right)$	(1, 6, 7, 1)	8	3.706
47	$\frac{32}{85}$	$\left(\frac{-165}{4573}, \frac{-20158}{32011}\right)$	(9051, 142546, 264089, 33059)	$\frac{1262}{987}$	24.724
51	$\frac{-73}{121}$	$\left(\frac{835}{1143}, \frac{-1298}{1143}\right)$	(129205, 145309, 168674, 23303)	$\frac{-553}{1283}$	22.230

Table 1: 20 concrete primitive solutions spanning Height 1–29 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \le y \le z)$	k	height(k)
53	$\frac{-8}{9}$	$\left(\frac{-1}{2}, \frac{-45}{14}\right)$	(28, 217, 453, 53)	$\frac{-11}{7}$	11.665
57	$\frac{4}{17}$	$\left(\frac{9}{22}, \frac{-1}{22}\right)$	(1228, 1871, 4513, 507)	$\frac{-225}{67}$	16.596
59	$\frac{53}{345}$	$\left(\frac{6408}{9377}, \frac{3521}{9377}\right)$	(228599, 398665, 545334, 63949)	$\frac{6419}{7003}$	27.057
61	$\frac{-40}{41}$	$\left(\frac{4049}{18937}, \frac{-41652}{18937}\right)$	(4, 5, 9, 1), (49126165667392699, 4069697547809175220, 10497171197304442521, 1136515825909764169), (44086175543016325800589 16, 15101250336932945896869775, 31551572519670198048461181, 34410504801045500059 75231)	$\frac{73}{17}$, $\frac{-208139518197}{9838278563}$, $\frac{-133479529624435}{86474661603029}$	4.443, 87.618, 117.458
67	$\frac{7}{97}$	$\left(\frac{-107}{233}, \frac{-43986}{31921}\right)$	(228599, 398665, 545334, 63949), (76790401669507993640525954196312 6011, 34726859248344919166576176143153 55158, 45898197506046998558026974605102 62963, 50622077135562418489381038035719 2673)	$\frac{31252757}{33079883}$, $\frac{306660869065034607000}{309215004168505028383}$	50.326, 50.326
69	$\frac{5}{169}$	$\left(\frac{182033}{581575}, \frac{379548}{581575}\right)$	(2512, 9347, 13145, 1409)	$\frac{-727}{343}$	18.306
71	-1	$(1, -2)$	(1, 3, 10, 1), (4355792802787, 134701573232361, 364707730611230, 36564540433213)	1, $\frac{7999986}{3124913}$	3.466, 65.923
73	$\frac{-4}{5}$	$(2, -3)$	(17139388, 18518399, 27476817, 2919107)	$\frac{-3401}{815}$	34.463
77	$\frac{28}{53}$	$\left(\frac{-1379}{192144}, \frac{-62735}{192144}\right)$	(111417913176, 652385155333, 3074200685543, 294747082303)	$\frac{577723}{2579259}$	58.008
79	$\frac{-4}{5}$	$(2, -3)$	(144169, 173690, 250323, 25489), (277675, 608674, 1725063, 163861)	$\frac{-405}{68}$, $\frac{469}{612}$	25.064, 28.855
83	$\frac{-8}{29}$	$\left(\frac{19}{27}, \frac{20}{27}\right)$	(222601, 2408274, 2863999, 292549)	$\frac{2087}{1741}$	31.655
89	$\frac{-4}{9}$	$\left(\frac{1}{124}, \frac{-183}{124}\right)$	(1741159879, 278196472772, 415156380825, 38743789163)	$\frac{-1944381}{9383}$	51.797
91	$\frac{-9}{41}$	$(9, 10)$	(1, 9, 10, 1), (138503093432173, 329381955044706, 376645726192885, 37359362719357)	1, $\frac{8385371}{6909763}$	3.900, 67.147
93	$\frac{-56}{65}$	$\left(\frac{4}{7}, \frac{-11}{7}\right)$	(4, 7, 11, 1)	-1	4.381
97	$\frac{29}{193}$	$\left(\frac{1188069}{498019}, \frac{-403076}{498019}\right)$	(10375142630827583852468, 33831167245789295195 7817, 64147164755281430412 1567, 55798826475782554871307), (15074456415583166678369440 52983, 25264402158620894763817067 31988, 49790928437149013096941809 26497, 43284024259490699517752278 4739)	$\frac{-102102687859481}{229822817432650}$, $\frac{-19024649582181125}{131231457030930976}$	108.933, 141.009

Table 2: 23 concrete primitive solutions spanning Height 1–37 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \leq y \leq z)$	k	height(k)
101	$\frac{64}{585}$	$\left(\frac{210919}{356203}, \frac{13332}{356203}\right)$	(115665246982780081781754947091286365446352306 99, 55533467549319684604321403836601861318197410455, 96201694481007146665176733936502346101197249772, 82645193175620457358519148778990654095326341 89)	$\frac{49705409379188979478965 33}{33171077050940981350902139}$	218.043
103	$\frac{8}{97}$	$\left(\frac{25953}{40597}, \frac{2456}{40597}\right)$	(1891967, 5736274, 9624139, 821739)	$\frac{4155}{16021}$	32.897
107	-1	$(1, -2)$	(2407, 7143, 15026, 1237)	$\frac{5}{32}$	18.106
109	$\frac{-3}{5}$	$\left(\frac{-75}{103}, \frac{-284}{103}\right)$	(75, 103, 284, 23)	$\frac{-71}{7}$	9.913
111	$\frac{11}{61}$	$\left(\frac{-230291}{336901}, \frac{-492790}{336901}\right)$	(1, 10, 11, 1)	$\frac{-10}{9}$	4.053
113	$\frac{4}{5}$	$\left(\frac{-1}{2}, \frac{-1}{2}\right)$	(7, 7, 12, 1), (223937, 237668, 312753, 27809)	1, $\frac{679}{817}$	5.565. 26.073
119	$\frac{-133}{157}$	$\left(\frac{-506251}{59374565}, \frac{-130362878}{59374565}\right)$	(30287965481668073, 434289126261707635, 996472927107738074, 77496631489680909)	$\frac{-5229646070084}{2135357527917}$	82.860
123	$\frac{32}{37}$	$\left(\frac{-9}{22}, \frac{1}{22}\right)$	(10778, 13981, 20717, 1671)	$\frac{-159}{89}$	19.127
127	$\frac{3}{61}$	$\left(\frac{-49}{279}, \frac{314}{279}\right)$	(669, 706, 1481, 113), (21336356794, 44048595009, 73294947149, 5648260099), (712189021550467442073, 445101056961517132 5278, 543857824525894446577 3, 445233454211910375263)	$\frac{-166}{65}$, $\frac{-275905}{256811}$, $\frac{52895327830}{80639326543}$	14.877, 50.023, 100.946
129	$\frac{65}{97}$	$\left(\frac{-3225}{19201}, \frac{-808}{2743}\right)$	(5, 8, 13, 1)	$\frac{-21}{5}$	4.105
131	-1	$(1, -2)$	(218031639, 1068400313, 3263827490, 240478223)	$\frac{22043}{10028}$	42.674
133	$\frac{-7}{13}$	$\left(\frac{5}{3}, \frac{92}{51}\right)$	(443, 689, 736, 63), (148235522557, 298646264321, 680583220672, 50105570931)	$\frac{281}{391}$, $\frac{-160955}{732989}$	12.119, 51.843
137	$\frac{56}{181}$	$\left(\frac{1309}{4936}, \frac{-1315}{4936}\right)$	(28988, 107079, 369559, 26597)	$\frac{-15311}{18473}$	28.398
139	$\frac{3}{109}$	$\left(\frac{128047}{191015}, \frac{49018}{191015}\right)$	(292642, 821065, 1206057, 90373)	$\frac{-29699}{22174}$	28.063
141	$\frac{-60}{61}$	$\left(\frac{512045}{261178}, \frac{-964513}{261178}\right)$	(72211175, 446853184, 1030134149, 73587999)	$\frac{-19014301}{5702545}$	40.428
143	-1	$(1, -2)$	(3, 7, 14, 1), (1233, 22603, 62174, 4391)	0, $\frac{-42}{59}$	4.152, 20.938
149	$\frac{-19}{85}$	$\left(\frac{-929}{1733}, \frac{3156}{1733}\right)$	(636, 835, 977, 77)	$\frac{236}{383}$	12.442
151	$\frac{-45}{53}$	$\left(\frac{5}{9}, \frac{14}{9}\right)$	(5, 9, 14, 1)	1	4.188

Table 3: 23 concrete primitive solutions spanning Height 1–47 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \le y \le z)$	k	height(k)
157	$\frac{-3}{17}$	$\left(\frac{141}{95}, \frac{88}{95}\right)$	(31658744, 45620973, 54530839, 4118141), (1110466451945907112, 1796596501774612643, 1893976757448252081, 149773454264774017)	$\frac{15661}{19472}$, $\frac{1624110418}{2265840721}$	34.856, 83.460
159	$\frac{4}{29}$	$\left(2, \frac{-3}{7}\right)$	(337729, 353654, 622355, 43357)	$\frac{-1747}{2359}$	25.795
161	$\frac{77}{81}$	$\left(\frac{-2063}{3319}, \frac{36}{3319}\right)$	(262692, 641495, 842767, 60149)	$\frac{-292}{507}$	24.866
163	$\frac{-1}{17}$	$\left(\frac{2141}{2881}, \frac{-1158}{2881}\right)$	(99233, 394727, 518634, 36731)	$\frac{-1532}{1285}$	26.001
167	$\frac{4}{17}$	$\left(\frac{9}{22}, \frac{-1}{22}\right)$	(74, 673, 787, 57), (1858416522, 7036228109, 12127154449, 810685471)	$\frac{-24}{29}$, $\frac{124174}{198221}$	13.751, 47.606
173	$\frac{8}{9}$	$\left(\frac{-7}{5}, \frac{24}{35}\right)$	(63816, 94031, 110677, 7997)	$\frac{-257}{42}$	20.166
177	$\frac{1}{5}$	$\left(\frac{11}{25}, \frac{-24}{175}\right)$	(1897, 2681, 3916, 263), (30650011, 163560484, 247875733, 16362853)	$\frac{313}{301}$, $\frac{52873}{73871}$	14.498, 36.275
179	$\frac{-112}{145}$	$\left(\frac{17337}{532423}, \frac{-1046582}{532423}\right)$	(586876415, 2789777249, 4823616902, 311333271)	$\frac{28727529}{1382477}$	44.395
181	$\frac{-7}{13}$	$\left(\frac{5}{3}, \frac{92}{51}\right)$	(6053819, 6053819, 18445888, 1159779)	$\frac{-743}{1649}$	30.808
183	$\frac{12}{61}$	$\left(\frac{-479}{1970}, \frac{-2051}{1970}\right)$	(18204554218, 27852627509, 62090414753, 3904986579), (446105853670800384 53, 280441072461263875439, 614193319048740170854, 38587307753389480293)	$\frac{-1396613}{10438518}$, $\frac{142405631833}{89470775462}$	48.163, 93.957
187	$\frac{84}{197}$	$\left(\frac{467}{12540}, \frac{5717}{12540}\right)$	(6086, 9507, 10861, 761)	$\frac{-4261}{5129}$	20.390
191	$\frac{7}{17}$	$\left(\frac{1}{5}, \frac{6}{115}\right)$	(6, 23, 115, 7), (55775, 4076738, 7487949, 465299), (10850077656912535565, 24303123712485044202, 80227385961925004089, 4892095537343672191)	−13, $\frac{241}{443}$, $\frac{300712263}{988868693}$	6.553, 30.058, 88.798
193	$\frac{-4}{37}$	$\left(\frac{7}{12}, \frac{7}{12}\right)$	(110056310545452804845699037308569120 4, 7382952310363167848731758733032453061, 9311364230149643442162588460858110829, 6125689926695106868495247590541574 87)	$\frac{621566628330181837}{501141307606379545}$	170.445
197	$\frac{-188}{137}$	$\left(\frac{-4538}{19963}, \frac{209465}{19963}\right)$	(50704, 298617, 012059, 180461)	$\frac{-364547}{1019}$	31.740
201	$\frac{32}{49}$	$\left(\frac{-178}{291}, \frac{-245}{291}\right)$	(111600681784985396 2048, 200446234533102675 4775, 356451448737315466 3031, 216988690452227767853), (26302199512282543160242598 06608, 5225239243632102563759659890625, 1953321277223705540911787 0750351, 1160134160384542904517035352013)	$\frac{-82436541535}{80695310527}$, $\frac{1854461647968235}{11040287486584853}$	98.846, 143.527

Table 4: 24 concrete primitive solutions spanning Height 1–37 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \le y \le z)$	k	height(k)
209	$\frac{4}{17}$	$\left(\frac{9}{22}, \frac{-1}{22}\right)$	(456741325525, 11021276613067, 13918037099988, 879530604163)	$\frac{4210157}{2492579}$	60.758
211	$\frac{12}{181}$	$\left(\frac{2050}{4697}, \frac{2197}{4697}\right)$	(50655913495, 105827561137, 169655836818, 10191152909), (14003920161371413178738760326673137187432108473, 14532860738109082580035239647893221943634610137, 31605892974868731202618662984289465373267063590, 18665984223522670272685161301071869832411 68721)	$\frac{-20060493}{22852307}$, $\frac{-98804180223947814399088 7}{82609494217775183224996 7}$	54.022, 212.894
213	$\frac{-96}{85}$	$\left(\frac{2859}{2033}, \frac{6602}{2033}\right)$	(46410580396087, 48706449849644, 118968414820139, 6940828958469)	$\frac{546386203}{285025889}$	66.220
217	$\frac{-39}{89}$	$\left(\frac{-991}{35023}, \frac{52880}{35023}\right)$	(3, 3, 16, 1)	$\frac{-201}{19}$	4.365
219	$\frac{-20}{29}$	$\left(\frac{5}{2}, \frac{7}{2}\right)$	(6905383801370, 10082271789347, 12674696250587, 795508575341)	$\frac{1003801}{812349}$	60.080
221	$\frac{-7}{13}$	$\left(\frac{5}{3}, \frac{92}{51}\right)$	(18170075829628946534152326312538321309945, 32469491979966057952627731170720804388 0141, 54594243497875426113235638376549635732 7888, 31804802854316895728509269912421630763669)	$\frac{4554725986626873439777}{2251181595697275141261}$	190.309
223	$\frac{-64}{53}$	$\left(\frac{299}{487}, \frac{-1356}{487}\right)$	(386, 1123, 3489, 197)	$\frac{-109}{5}$	17.397
227	-1	$(1, -2)$	(31731, 855269, 2323538, 130273)	$\frac{613}{887}$	28.180
229	$\frac{72}{181}$	$\left(\frac{3383}{29435}, \frac{1584}{4205}\right)$	(147763814238656858860, 608999264562929036617, 1499093353525846395453, 83865054397121607863)	$\frac{-2055494927143}{1138143155147}$	99.818
231	$\frac{28}{29}$	$\left(\frac{-354}{365}, \frac{131}{365}\right)$	(5389, 98690, 145231, 14889), (71207, 202210, 244043, 14889)	$\frac{75}{91}$, $\frac{1367}{47927}$	22.388, 23.560
233	$\frac{4}{53}$	$\left(\frac{-4507}{888}, \frac{-5047}{888}\right)$	(1103010172, 1861529873, 3269841633, 185225597)	$\frac{413383}{450883}$	43.493
237	$\frac{-104}{185}$	$\left(\frac{-31669}{19548}, \frac{-79271}{19548}\right)$	(4, 13, 17, 1)	$\frac{-1}{25}$	5.443
239	$\frac{136}{261}$	$\left(\frac{29531}{484199}, \frac{71760}{484199}\right)$	(1640984772620303606, 3117491002540389405, 3477412467901354591, 215837293427890067)	$\frac{-890704612273}{634303599357}$	85.474
241	$\frac{-15}{113}$	$\left(\frac{5490053}{9654187}, \frac{-6040624}{9654187}\right)$	(1, 15, 16, 1)	$\frac{2413}{2747}$	4.635
249	$\frac{-24}{145}$	$\left(\frac{-73441}{147}, \frac{12272}{21}\right)$	(7790928430539881312, 5953102340249721 4735, 6168923790535940 7721, 3842966957265123567)	$\frac{-49547964571}{43598116307}$	93.180
251	$\frac{7}{9}$	$\left(\frac{-137}{187}, \frac{-138}{187}\right)$	(1403, 1530, 3109, 169)	$\frac{188}{43}$	14.875
253	$\frac{32}{37}$	$\left(\frac{-9}{22}, \frac{1}{22}\right)$	(110317, 142927, 204068, 11577)	$\frac{-333}{467}$	23.913

Table 5: 19 concrete primitive solutions spanning Height 1–47 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \le y \le z)$	k	height(k)
263	$\frac{35}{117}$	$\left(\frac{6697}{23029}, \frac{5298}{23029}\right)$	(4775149057432683646382312024339, 3539427233368322885773144 8412866, 4419093367672188369709369 0461217, 2497644678140975819970225230281)	$\frac{52149128491363779}{20175058620871894}$	146.247
267	$\frac{7}{17}$	$\left(\frac{1}{5}, \frac{6}{115}\right)$	(25731556254637, 37497158826659, 44825895328366, 2593731455009)	$\frac{-14374358}{11142511}$	61.575
281	$\frac{40}{53}$	$\left(-3, \frac{-8}{7}\right)$	(7712425, 9015359, 21799904, 1105641)	$\frac{40}{53}$	33.682
287	$\frac{-16}{13}$	$\left(\frac{59}{6}, \frac{-229}{6}\right)$	(171944734111, 1109428229629, 4149623533526, 206235722667)	$\frac{-6508897}{1568225}$	55.732
291	$\frac{95}{193}$	$\left(\frac{9241}{331533}, \frac{112798}{331533}\right)$	(5, 14, 19, 1)	$\frac{137}{167}$	5.174
293	$\frac{8}{97}$	$\left(\frac{25953}{40597}, \frac{2456}{40597}\right)$	(1412817, 4957627, 5941544, 322369)	$\frac{-12029}{9770}$	31.648
299	$\frac{4}{29}$	$\left(2, \frac{-3}{7}\right)$	(4323, 4886, 11725, 577)	$\frac{23}{7}$	17.785
301	$\frac{8}{25}$	$\left(\frac{-151}{14}, \frac{115}{14}\right)$	(1, 40, 59, 3), (3197413, 8989336, 20127265, 985251)	1, $\frac{-30871}{43497}$	5.277, 32.139
303	$\frac{43}{53}$	$\left(\frac{-791}{809}, \frac{654}{809}\right)$	(60418, 64147, 131801, 6521)	$\frac{-799}{1121}$	23.744
307	$\frac{-17}{145}$	$\left(\frac{1}{17}, \frac{18}{17}\right)$	(1, 17, 18, 1)	1	5.500
309	$\frac{-16}{13}$	$\left(\frac{59}{6}, \frac{-229}{6}\right)$	(5053724375353144911988872935972345 5748, 1115386020862704657707036079177927 74669, 3491495668822635825877607158619782 88615, 1674737939290130874084488964823522 9551)	$\frac{-240428351310879884753}{546111739682315 81989}$	177.300
311	$\frac{4}{9}$	$\left(\frac{1}{22}, \frac{9}{22}\right)$	(480499, 1460127, 1843850, 95599), (20564342260380338, 39547794771476995, 40541662859715111, 2290727287853597)	$\frac{-404}{323}$, $\frac{314529827}{124003321}$	29.287, 77.168
313	$\frac{-4}{37}$	$\left(\frac{7}{12}, \frac{7}{12}\right)$	(63617, 256468, 49847, 17721), (3576723341261638039, 1913099699533775243 9, 2045266070227271767 6, 1120715103354923061)	$\frac{547}{415}$, $\frac{111223625}{29913404}$	25.711, 89.098
317	$\frac{1}{5}$	$\left(\frac{11}{25}, \frac{-24}{175}\right)$	(7, 8, 21, 1)	$\frac{1}{2}$	3.585
321	$\frac{-4}{25}$	$\left(\frac{-61}{4}, \frac{-75}{4}\right)$	(58331413, 285600212, 392809795, 19609631), (174178388755, 197422814243, 290663175068, 14682502459)	$\frac{13013}{5287}$, $\frac{1562101}{1461959}$	39.599, 52.296
323	$\frac{23}{169}$	$\left(\frac{2615}{1183}, \frac{-822}{1183}\right)$	(5200334, 6915127, 15651159, 741337), (10056391537, 10489425578, 25649634201, 1215303869)	$\frac{-48355}{83266}$, $\frac{-1756609}{2893876}$	33.217, 48.099
327	$\frac{-8}{17}$	$(4, -5)$	(345979275326, 944763892877, 1083164953879, 56553208021), (28517623616471761, 480168116623739902, 696543853973747909, 34081978453197269)	$\frac{-1124197}{763369}$, $\frac{125813767}{54890199}$	56.891, 83.576

Table 6: 23 concrete primitive solutions spanning Height 1–39 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \leq y \leq z)$	k	height(k)
329	$\frac{4}{9}$	$\left(\frac{1}{22}, \frac{9}{22}\right)$	(480499, 1460127, 1843850, 95599), (20564342260380338, 39547794771476995, 40541662859715111, 2290727287853597)	$\frac{-404}{323}$, $\frac{314529827}{124003321}$	29.287, 77.168
331	$\frac{-12}{61}$	$\left(\frac{1061}{5602}, \frac{5665}{5602}\right)$	(6597, 15346, 25555, 1219), (541857066796020771, 885401228992685435, 1150566324558260578, 57844544319727019)	$\frac{886}{205}$, $\frac{-52264555230}{8263319543}$	22.622, 85.266
337	$\frac{-8}{29}$	$\left(\frac{19}{27}, \frac{20}{27}\right)$	(4776835894317290339252315 71, 609155889204579017085296789, 646505803243611373489980464, 355909657159437067132251 99)	$\frac{-124757908803467}{29257222028207}$	124.443
339	$\frac{7}{17}$	$\left(\frac{1}{5}, \frac{6}{115}\right)$	(789002617798315, 3216798294345842, 8338185383712619, 382914281366557), (194540890552586868 3715, 196607766052497342 7466, 109171258341860878 51003, 498855109999502674849)	$\frac{27976397}{72066797}$, $\frac{-127083629023}{21660021221}$	71.463, 99.538
341	$\frac{56}{153}$	$\left(\frac{9487}{703}, \frac{4740}{703}\right)$	(314526385876749971933, 1730643343498146656 7465, 2823162311916369824 5348, 1328749814282166736751), (478005925829323518609 75733, 585884468939029229639 13068, 791245418841358726060 36215, 394274813274196688893 8421)	$\frac{1830653404979}{3361422194475}$, $\frac{30937862242765}{82111824797237}$	102.896, 119.130
347	-1	$(1, -2)$	(1, 9, 22, 1)	$\frac{1}{2}$	5.047
349	$\frac{-51}{149}$	$\left(\frac{-515019}{2097679}, \frac{-3442940}{2097679}\right)$	(3, 17, 20, 1)	$\frac{-17}{20}$	4.959
353	$\frac{-80}{101}$	$\left(\frac{-1847}{51171}, \frac{-107938}{51171}\right)$	(56826587, 67285064, 96372157, 4656633)	$\frac{138607}{180287}$	38.478
357	$\frac{21}{29}$	$\left(\frac{-1051}{367}, \frac{500}{367}\right)$	(517928533, 645833876, 931323553, 44485487), (10982521940339, 13497568599092, 18893684514431, 910483289317), (14604134202668, 14836699500811, 70728669041791, 3150726443759)	$\frac{-132254}{54009}$, $\frac{-1163248}{4097349}$, $\frac{4992791}{25135879}$	39.714, 59.575, 61.870
359	$\frac{4}{17}$	$\left(\frac{9}{22}, \frac{-1}{22}\right)$	(181, 246, 695, 31), (169472648395, 1686703476847, 2475962075038, 115376690973)	$\frac{-2}{7}$, $\frac{-911938}{427483}$	13.977, 57.264
367	$\frac{-12}{29}$	$\left(\frac{18}{49}, \frac{-53}{49}\right)$	(443726853223, 1789770309494, 2123612069147, 103269591747)	$\frac{-6614343}{6546359}$	56.392
371	$\frac{-8}{17}$	$(4, -5)$	(778, 835, 837, 47), (2466759, 2652505, 2661446, 149279)	$\frac{-27}{13}$, $\frac{-743}{553}$	13.731, 29.858

Table 7: 21 concrete primitive solutions spanning Height 1–26 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \le y \le z)$	k	height(k)
373	$\frac{20}{29}$	$\left(\frac{-4768}{25381}, \frac{417}{1493}\right)$	(14586577187725577177096, 23096846931152339602833, 46172621049761681632307, 20458768365255165 61477), 	$\frac{-2422871858017}{5531856449947}$,	105.036,
			(11947663069831666526978 6439547, 1683759350326748608713 47943207, 2454967534122976339877 01496216, 1136353599575587939088 5493333)	$\frac{19138979793030483}{21160213894959292}$	136.176
379	$\frac{-12}{29}$	$\left(\frac{18}{49}, \frac{-53}{49}\right)$	(272369018, 1367002545, 2299179863, 102282073),	$\frac{-257819}{52859}$,	44.863,
			(1184709513607649570, 1217418399461938189, 1336992656954547211, 71149235234846499)	$\frac{7145078099}{2578550851}$	83.114
383	$\frac{204}{377}$	$\left(\frac{-12212}{64707}, \frac{-38035}{64707}\right)$	(522712745975867496520468 70748358, 5644218259977757556184964 0531969, 6803716501521216241039873 8968939, 3396464828358953323939105 317063)	$\frac{1550408297639667679}{1935402453467912411}$	149.093
389	$\frac{989}{232}$	$\left(\frac{29825}{190557}, \frac{31732}{190557}\right)$	(279022051747147185399480 8398379, 3375566679888983646972955 355900, 3963453138704995482589945 354391, 1949583280428270444711671 36157)	$\frac{59056815385513828}{47189577215495093}$	139.722
393	$\frac{56}{73}$	$\left(\frac{-209}{863}, \frac{-36}{863}\right)$	(201277984, 2153997596, 3061641247, 142059643)	$\frac{-127457}{146606}$	41.032
397	$\frac{16}{169}$	$\left(\frac{-19}{1852}, \frac{1695}{1852}\right)$	(161702491626027642898883 639631, 4872124138043623181479742 12348, 8442614453139414405033783 54043, 3659108096062685671514684 7589)	$\frac{-1301422837948593}{1818998862358337}$	138.870
401	$\frac{-16}{37}$	$\left(\frac{-7}{3}, \frac{14}{3}\right)$	(4707730316011215672320120149359167312, 941334729461875864112918220776962 7475, 1971053221347856842928278241378256 1773, 8388999288218105980257171919403689 01),	$\frac{-4704908635406554703}{3352465208783919657}$,	173.696,
			(275479357776418670537852824303948932 09881073, 557004444722179796858034985001103605 64919360, 563575497513161377150225747644915302 90180993, 281831441141048226662777953820533630 2056719)	$\frac{-32772666576884249 63381}{54460645468769601 85097}$	203.585
403	$\frac{3}{5}$	$\left(\frac{-1}{3}, \frac{2}{3}\right)$	(9, 38, 167, 7),	$\frac{-11}{7}$,	9.536,
			(19, 86, 549, 23),	$\frac{2}{11}$,	11.899,
			(22619, 32386, 34053, 1699)	$\frac{44}{9}$	11.899
407	$\frac{35}{61}$	$\left(\frac{-83}{93}, \frac{106}{93}\right)$	(232344524219, 424051131622, 526551287113, 24119392479),	$\frac{-3178226}{737263}$,	52.289,
			(8509742099900499, 14433696294090874, 16710655818075457, 786275362671487)	$\frac{-50016137}{9567037}$	74.412

Table 8: 15 concrete primitive solutions spanning Height 1–44 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \leq y \leq z)$	k	height(k)
971	$\frac{-16}{13}$	$\left(\frac{59}{6}, \frac{-229}{6}\right)$	(83653717, 247059774, 1300882915, 35116733),	$\frac{-147733}{45674}$,	43.009,
			(3477594175419544564438607, 4453246382814222024556694, 13178994314491788157140905, 3572233708833465206408831)	$\frac{-3667798345571}{807333349114}$	115.623
977	$\frac{-16}{13}$	$\left(\frac{59}{6}, \frac{-229}{6}\right)$	(38226631, 282173544, 1069737841, 28813573)	$\frac{-18613}{5251}$	42.618
979	$\frac{24}{149}$	$\left(\frac{-67963}{1570773}, \frac{1383620}{1570773}\right)$	(132603685, 255885862, 558892883, 15194199)	$\frac{-8733377}{6799957}$	37.703
			(9403119219230345885, 11596111087396501294, 28312245627197930451, 768450793389517267)	$\frac{-58062026215}{2061994855001}$	88.171
983	$\frac{-5}{117}$	$\left(\frac{-45383}{464953}, \frac{530466}{464953}\right)$	(452500536169517516094, 823742386103319270269, 1160078784327577806233, 33077493446993186419)	$\frac{-4735382136040}{2498221740549}$	96.623
989	$\frac{-104}{81}$	$\left(\frac{103}{346}, \frac{1413}{346}\right)$	(276706435, 471788428, 1690784721, 45286241),	$\frac{282729}{196337}$,	42.583,
			(16483380987870004461644, 16805632045172389358375, 63642801016634531526477, 1705710645024622106587)	$\frac{-352502172041}{30594988227}$	105.102
997	$\frac{4}{121}$	$\left(\frac{-5921}{10634}, \frac{16137}{10634}\right)$	(22457303375602142923450437370936871278611663494196843, 265496357325318674335897812161139446452263143557538584, 349822605324090823518870097900624087451082461316401009, 10008104623522696964064410352949916558972840407413287)	$\frac{-538285840455706011951086061}{447779212309641648847058891}$	244.611
1001	$\frac{13}{17}$	$\left(\frac{-397}{1673}, \frac{16}{1673}\right)$	(12653155529645773041, 14496165953243782820, 30632337876005776777, 829827057251813143)	$\frac{6864432341}{12975328069}$	87.685
1003	$\frac{-16}{65}$	$\left(\frac{-48448}{308665}, \frac{-62703}{44095}\right)$	(357153561, 441486806, 522795199, 15911377)	$\frac{1220947}{2671121}$	39.595
1007	$\frac{-220}{157}$	$\left(\frac{321}{60268}, \frac{973057}{60268}\right)$	(18754633151703759081272317852581751270809408588853, 24040798893695666175180959976981448725994130618754, 276962098995579830477463625832507108420988928415519, 7339332340035270076957220087297502594586408542833)	$\frac{-246903836004572641662901577}{6914579901675743188735327}$	232.595
1013	$\frac{3}{101}$	$\left(\frac{20951}{17421}, \frac{5908}{17421}\right)$	(24574653502948757745404, 98778234488177851314697, 101516509859021233400459, 3148857129793207750683)	$\frac{9963327853555}{2964128052389}$	104.165

Table 9: 13 concrete primitive solutions spanning Height 8–54 digits. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

Appendix 3: Other Solutions for $x^4 + y^4 + z^4 = 2n^2w^4$ $(n = 3, 13, 17, 31, 41, 47, 51)$

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \le y \le z)$	k	height(k)
3	$\frac{3}{37}$	$\left(\frac{5053}{7821}, \frac{-190}{7821}\right)$	(191, 758, 1039, 537)	$\frac{-667}{409}$	13.646
3	$\frac{4}{5}$	$\left(\frac{-1}{2}, \frac{-1}{2}\right)$	(1, 1, 2, 1), (191, 199, 278, 149)	$\frac{1}{3}$, $\frac{-11}{2}$	1.630, 11.704
3	$\frac{-8}{197}$	$\left(\frac{327}{784}, \frac{71}{112}\right)$	(4430505880486317556231058410183907705265377, 15091518989598701515761230465623349480831602, 15687731329451834930350105762440616394266249, 8897807748791604925680134889341966921091629)	$\frac{-1989698119095368204817}{3084386303584566632969}$	198.025
3	$\frac{12}{49}$	$\left(\frac{3036}{1153}, \frac{-455}{1153}\right)$	(91, 109, 326, 159), (93163, 251938, 659723, 322011)	$\frac{-16}{13}$, $\frac{-999}{19}$	10.380, 25.542
3	$\frac{24}{89}$	$\left(\frac{91}{326}, \frac{-109}{326}\right)$	(91, 109, 326, 159), (4757935328932114, 7041425736632701, 18624544273367213, 9097367279997867)	$\frac{109}{235}$, $\frac{-304072675}{964918062}$	12.680, 76.064
3	$\frac{28}{29}$	$\left(\frac{-354}{365}, \frac{131}{365}\right)$	(191, 758, 1039, 537), (799, 86614, 127823, 65097)	$\frac{-201}{89}$, $\frac{-561}{449}$	13.750, 23.330
3	$\frac{-28}{37}$	$\left(\frac{993}{2230}, \frac{3271}{2230}\right)$	(167561, 356822, 517039, 264773)	$\frac{1189}{1237}$	28.056
13	$\frac{-7}{13}$	$\left(\frac{5}{3}, \frac{92}{51}\right)$	(13093, 56009, 107168, 25449)	$\frac{-95}{1649}$	20.523
17	$\frac{-8}{29}$	$\left(\frac{19}{27}, \frac{20}{27}\right)$	(9537, 14972, 18217, 4133), (114791, 152036, 193919, 43773), (20433473, 21407897, 28928592, 6581821)	$\frac{-86}{191}$, $\frac{-146}{377}$, $\frac{5057}{11731}$	19.265, 23.212, 35.988
17	$\frac{-7}{157}$	$\left(\frac{10141}{7671}, \frac{-3688}{7671}\right)$	(201, 521, 748, 161)	$\frac{2435}{2941}$	13.347
31	$\frac{12}{37}$	$\left(\frac{-1355}{21194}, \frac{-15377}{21194}\right)$	(1, 5, 6, 1), (18491070898521258, 93341526748788265, 112278484660039843, 18698442145369807), (393556685397950, 58065700817685469, 88218078555075633, 13908980781287987)	$\frac{12}{37}$, $\frac{-10501431191}{679620377}$, $\frac{20664848231}{888156995}$	3.901, 78.835, 79.755

Table 10: 19 concrete primitive solutions. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \leq y \leq z)$	k	height(k)
41	$\frac{17}{81}$	$\left(\frac{79}{1393}, \frac{-144}{199}\right)$	(9377628687576699475323058281586350049259392, 1343001470458991003015885097917300059610089 5, 2030328175647195043095853213441635438238436 9, 2811934334922047505814776216556178874202477)	$\frac{-13540128744306617132469 6}{43993860266701330425262 1}$	196.933
41	$\frac{44}{169}$	$\left(\frac{-3998}{253}, \frac{3185}{253}\right)$	(5441389686377966197, 5952881649156946338 1, 7370607112236248198 0, 1057663780912317296 7)	$\frac{-36225655093}{42531504399}$	90.534
41	$\frac{52}{137}$	$\left(\frac{391}{4038}, \frac{-1787}{4038}\right)$	(92988, 185585, 200711, 30433)	$\frac{16427}{15371}$	27.266
41	$\frac{-56}{89}$	$\left(\frac{24903}{68491}, \frac{92168}{68491}\right)$	(5431823739762307870494663376411509084524164558586588 5, 9311318754901349111904285274211583718864898597015156 8, 2421447289522576248114036720335024228392322029821462 9, 3199209295879800905879366515891137451157169722911518 7)	$\frac{-30642294010246440479709621005}{23123774046491324438403430133}$	247.459
47	$\frac{-56}{65}$	$\left(\frac{4}{7}, \frac{-11}{7}\right)$	(839990066, 10754417721, 25232397949, 3120163151), (1397258788546783720583 71, 6658679423595281165716 74, 1271597488901889913049 231, 1588287609865245328205 81)	$\frac{-82553}{13687}$, $\frac{970150063231}{1037070568423}$	47.729, 47.729
47	$\frac{-77}{101}$	$\left(\frac{-73329}{562759}, \frac{1231694}{562759}\right)$	(1023004374308547787420341991609 49, 1121555880372128695035018116259 759, 2387007549683564413844112467948 854, 2962882810171866777041350446332 57)	$\frac{-125218492480171553 62}{81078825958116128 09}$	154.884
47	$\frac{-116}{173}$	$\left(\frac{36268}{70407}, \frac{90967}{70407}\right)$	(2320229827735239, 5387196649410614, 7263410412294109, 953679704196799)	$\frac{-895706486}{365922931}$	75.777
51	$\frac{-33}{41}$	$\left(\frac{4217}{37405}, \frac{71798}{37405}\right)$	(98750347889143, 215709962056415, 331778377250878, 40773262891589)	$\frac{-653193988}{65186793}$	67.429
51	$\frac{39}{89}$	$\left(\frac{127177}{768221}, \frac{71486}{768221}\right)$	(238031626, 1654352467, 9651492515, 1136699363)	$\frac{-179287}{4289259}$	46.408
51	$\frac{-48}{101}$	$\left(\frac{-37}{22}, \frac{-85}{22}\right)$	(1917336662159156761, 1797465161734559975 9, 2523264798727588147 0, 3146304878811925557)	$\frac{-7024052407}{8353816087}$	87.570
51	$\frac{168}{193}$	$\left(\frac{-64203}{148033}, \frac{-19520}{148033}\right)$	(128945, 173302, 268897, 33313)	$\frac{-64877}{22367}$	24.646

Table 11: 12 concrete primitive solutions. All n satisfy mod 5, 16, 29 conditions and $\gcd(x, y, z, w) = 1$.

Appendix 4: Integer Solutions for $x^4 + y^4 + n^2 z^4 = w^4$ $(n = 2, 5, 6, 10, 11, 13, 19, 23, 29, 30, 31, 51, 59, 237)$

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \leq y,\ 0 < z,\ 0 < w)$	k	height(k)
2	$\frac{-200}{159}$	$\left(\frac{-245964}{989981}, \frac{-529188}{989981}\right)$	(1478137457663315453842481281587062723218541 7, 2293940545152874952496494599038302158255400 4, 1360444689076608306306888290765704394433525 4, 2606638372333951207505033089855417658018984 9)	$\frac{-1022202550885715927 1836149}{4973820434222253843674925}$	208.958
5	$\frac{-200}{33}$	$\left(\frac{1098625}{8664814}, \frac{-1136795}{8664814}\right)$	(44372076633894, 77909041151165, 27822312777444, 86389766182919), (10273306668320686311812656725, 21672525695420775656916403894, 24603748441277284283090649300, 55360346418742274771588669519)	$\frac{721672547}{401678173}$, $\frac{419879665303795 36039}{6361540450159680921}$	72.357, 140.825
6	$\frac{-192}{95}$	$\left(\frac{-152715}{572866}, \frac{382125}{572866}\right)$	(1759454436368892298338450609525092850 0, 1759454436368892298338450609525092850 0, 4058489384580869469949943875587274382, 2279533276456931440702811142458599173 7)	$\frac{-42447984107442442717799}{598742268306690798 6295}$	180.413
10	$\frac{-120}{167}$	$\left(\frac{-45178}{479763}, \frac{102800}{159921}\right)$	(2612468166777199841253766754310149 79, 9953555699080153402717856131761559 30, 4217286496503328972095499035926011 86, 1427242172593177154199688515965361 079)	$\frac{-13590223858908586478961 3}{18832198430159765391590 9}$	175.448
10	$\frac{-180}{101}$	$\left(\frac{2554361}{17748406}, \frac{12600277}{17748406}\right)$	(802025385470172555485353175564749871760349036460499 11, 357875720105384695686304974638016716956142041933914 210, 118825016210387182261744237492875690521504689721131 194, 436733538504200184127285271853259310131966303073062 811)	$\frac{9335016125881092226499321663 5}{355853024008309067090066196 58}$	255.762
11	$\frac{-80}{7}$	$\left(\frac{10}{147}, \frac{1280}{8379}\right)$	(6317639, 41597960, 72383300, 240122361)	$\frac{-3069037}{66063}$	40.945

Table 12: 6 concrete primitive solutions.

n	u	(x_0, y_0)	(x, y, z, w) $(0 < x \leq y,\, 0 < z,\, 0 < w)$	k	height(k)
11	$\frac{-36}{55}$	$\left(\frac{-4527}{25222}, \frac{9459}{25222}\right)$	(4561002278236940, 9623234663831655, 9716792562424596, 32294034739143193),	$\frac{2624854481}{355032557}$,	83.346,
			(50716637842684804005 70, 565230792395573779 6025, 198010471747412278 4036, 771481097857401760 1787)	$\frac{-13615651970667}{1804991832116}$	107.943
13	$\frac{-168}{167}$	$\left(\frac{-29303}{115806}, \frac{14383}{38602}\right)$	(2659779635796170 5344, 330150939232825391 61, 283604549529153557 2, 361060786137998824 57)	$\frac{181084518879}{136695871229}$	98.033
19	$\frac{-120}{187}$	$\left(\frac{-8324}{60429}, \frac{-10140}{20143}\right)$	(22454117760, 281831029555, 145773881796, 641474481677)	$\frac{-386344005}{33608693}$	63.509
23	$\frac{-184}{63}$	$\left(\frac{118273}{533478}, \frac{27105}{177826}\right)$	(529342994422784252, 1113189054592962532, 356143066543690741, 1783792077677461137)	$\frac{-14234349988247}{2682915907067}$	88.046
29	$\frac{-80}{27}$	$\left(\frac{2312}{18213}, \frac{-3360}{6071}\right)$	(17000695412303018454244 7668195, 229056236830248164066404223600, 53461279335894866928179432392, 319788414125749287078267357213)	$\frac{-433729111279715965}{12752075910589837}$	141.448
29	$\frac{-44}{45}$	$\left(\frac{1829}{4934}, \frac{-1895}{4934}\right)$	(138670868471770441741544767611038174732996174674406103 5855, 166054830422706214306242943232564436748055030318871839 0718, 268651482245552449240991791970051552029926577500275067480, 198998575463618709526364502642466034374417693021854358 3843)	$\frac{-776428310933294405588905268054 3}{296801871417539064492734622939 9}$	270.817
30	$\frac{-96}{113}$	$\left(\frac{6576}{19133}, \frac{10440}{19133}\right)$	(224424583255652913984072985387164863756134 6631, 350821555685175669956147338102020607392055 9590, 740145036742509923447447610590786527546533770, 459791662008261086598716121309448009762750 6283)	$\frac{-914806885721490909106798 43}{550909735740698633221040353}$	217.879
31	$\frac{-173}{40}$	$\left(\frac{1726}{11471}, \frac{27160}{103239}\right)$	(307280663748646704317766225849368024 0, 311083443213467463425196155120966426 5, 103095559030516968381488938658409737 6, 596784126089901302207072555470204493 7)	$\frac{23236795241672582935}{58343697667468059939}$	170.044
51	$\frac{-20}{9}$	$\left(\frac{6318}{23281}, \frac{762}{23281}\right)$	(230, 735, 152, 1139)	$\frac{214177}{23323}$	19.474
59	$\frac{-120}{13}$	$\left(\frac{275353}{3048798}, \frac{-90753}{1016266}\right)$	(6087, 30320, 3420, 33913)	$\frac{57750}{135137}$	28.196
237	$\frac{-168}{73}$	$\left(\frac{-16516}{46957}, \frac{24188}{46957}\right)$	(13634372516342, 43136916615357, 2198558272084, 46829983967879)	$\frac{-47422523}{1553457}$	71.307

Table 13: 11 concrete primitive solutions.

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